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BUBBLE WAKE SOLIDS CONTENT IN THREE-PHASE FLUIDIZED BEDS

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Abstract—It is shown that existing equations for predicting the holdups of wakes behind bubbles in three-phase fluidized beds are not entirely satisfactory. A new model is then developed whereby the wake is treated as the sphere-completing volume of a spherical cap bubble, due allowance being made for hydrodynamic interactions between bubbles. The generalized wake equations of Bhatia & Epstein (1974) are applied to compute the ratio of solids holdup in the wakes to that in the remaining liquid of the bed. Using experimental data from the literature, a rational equation is then generated for predicting this ratio from measured variables, and a mechanism for wake solids entrainment is proposed which is consistent with this equation.

INTRODUCTION

The influence of the wakes behind the gas bubbles on several aspects of three-phase fluidized bed behaviour has been well demonstrated in the literature (Stewart & Davidson 1964; Østergaard 1965; Stewart 1965; Efremov & Vakhrushev 1970; Rigby & Capes 1970; Østergaard 1971; Bhatia 1972; Bhatia & Epstein 1974; El-Temtamy 1974; Kim 1974; Page & Harrison 1974; Darton & Harrison 1975; Baker *et al.* 1977). The magnitude and composition of such wakes have, however, been subjects of dispute and are still not known with any certainty. The present paper describes the methods which have been used to calculate wake holdups from experimental data, compares and assesses the available empirical correlations, and presents a new method based on the assumption that the wakes occupy the sphere-completing volumes behind spherical cap bubbles. The generalized wake equations of Bhatia & Epstein (1974) are then used to determine the solids content of the resulting wakes from experimental data on phase holdups. The cocurrent gas–liquid fluidized beds considered are those in which the solids are wetted and supported by the liquid, the gas flow thus constituting a perturbation of a liquid-fluidized bed.

HOW WAKE HOLDUPS HAVE BEEN DETERMINED

Wake holdups in three-phase fluidized beds have heretofore been estimated from experimental measurements of gas and solid holdups. It has commonly been assumed that the bed can be divided into a liquid-fluidized region, a gas bubble region and a bubble-wake region, and that the bubbles and their wakes travel at the same velocity. The principle differences amongst investigators have been with respect to their assumption of x, the ratio of solids holdup in the wake to solids holdup in the liquid-fluidized region. Thus Stewart & Davidson (1964) and most others (Efremov & Vakhrushev 1970; El-Temtamy 1974; Darton & Harrison 1975; Baker *et al.* 1977) assumed $x = 0$, while Østergaard (1965) and Kim (1974) took x as unity. Rigby & Capes (1970) used both extreme assumptions, while Bhatia & Epstein (1974) generalized this approach by writing equations which were applicable for any value of x.

The generalized wake equations necessary to determine the wake holdup ϵ_w from experimental measurements of phase holdups are here summarized. By definition

$$x = \frac{\epsilon''_{sw}}{\epsilon''_{sf}} = \frac{\epsilon''_{sw}}{1 - \epsilon''_{lf}}, \qquad [1]$$

where x is relative solids holdup, ϵ''_{sw} and ϵ''_{sf} represent the solids holdup in the wake and liquid-fluidized regions, respectively, and ϵ''_{lf} represents the liquid holdup in the latter region.

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The overall bed voidage, ϵ, is the sum of the total gas holdup, ϵ_g, and the total liquid holdup, ϵ_l, the latter of which is divided between the wake region and the liquid-fluidized region:

$$\epsilon = \epsilon_g + \epsilon_w(1 - \epsilon''_{sw}) + (1 - \epsilon_g - \epsilon_w)\epsilon''_{lf}. \qquad [2]$$

Combination of [1] and [2] gives

$$\epsilon = \epsilon_g + \epsilon_w(1 - x) + (1 - \epsilon_g - \epsilon_w + \epsilon_w x)\epsilon''_{lf}. \qquad [3]$$

The liquid velocity in the liquid-fluidized zone relative to the solids in this zone is given by (Bhatia & Epstein 1974)

$$v''_{ls} = \frac{j''_{lf}}{\epsilon''_{lf}} - v''_{sf} = \frac{j_l - v_g\epsilon_w(1 - x + x\epsilon''_{lf})}{(1 - \epsilon_g - \epsilon_w)\epsilon''_{lf}} + \frac{v_g\epsilon_w\epsilon''_{sw}}{(1 - \epsilon_g - \epsilon_w)\epsilon''_{sf}} \qquad [4]$$

where j_l and j''_{lf} denote superficial liquid velocities in the bed as a whole and in the liquid-fluidized zone, respectively; while v_g and v''_{sf} denote actual average gas velocity in the bed and solids velocity in the liquid-fluidized zone, respectively. By means of [1] and the relationship between v_g and superficial gas velocity, j_g,

$$v_g = j_g/\epsilon_g, \qquad [5]$$

[4] simplifies to

$$v''_{ls} = \frac{j_l - j_g\epsilon_w(1 - x)/\epsilon_g}{(1 - \epsilon_g - \epsilon_w)\epsilon''_{lf}}. \qquad [6]$$

The liquid volume fraction of the liquid-fluidized region, ϵ''_{lf}, is given by a Richardson–Zaki (1954) type equation,

$$\epsilon''_{lf} = \left(\frac{v''_{ls}}{v_1}\right)^{1/(n-1)} \qquad [7]$$

where n is the slope and v_1 the intercept at $\epsilon = 1$ when j_l in the absence of gas is plotted against ϵ on log–log coordinates. The parameters n and v_1 may be determined experimentally from expansion measurements on the liquid-fluidized bed, or they may be estimated for a given liquid–solid system and column diameter from the empirical equations of Richardson & Zaki (1954) or of Neuzil & Hrdina (1965).

If x is fixed, then experimental measurement of the independent variables j_l and j_g, and the dependent variables ϵ_g and $\epsilon(= 1 - \epsilon_s)$, allows solution of [3], [6] and [7] for the remaining dependent variables ϵ''_{lf}, v''_{ls} and ϵ_w. Putting $x = 0$, [3] reduces to

$$\epsilon = \epsilon_g + \epsilon_w + (1 - \epsilon_g - \epsilon_w)\epsilon''_{lf} \qquad [8]$$

and [6] to

$$v''_{ls} = \frac{j_l - j_g\epsilon_w/\epsilon_g}{(1 - \epsilon_g - \epsilon_w)\epsilon''_{lf}}. \qquad [9]$$

Here there is no solids upflow via the wakes and hence the solids downflow velocity in the liquid-fluidized zone, $-v''_{sf}$ of [4], is zero. Equations [8] and [9], together with [7], were used by Stewart (1965) and others (Efremov & Vakhrushev 1970; El-Temtamy 1974; Darton & Harrison 1975; Baker *et al.* 1977; Rigby & Capes 1970) to calculate ϵ_w. Putting $x = 1$, [3] reduces to

$$\epsilon = \epsilon_g + (1 - \epsilon_g)\epsilon''_{lf} \qquad [10]$$

and [6] to

$$v''_{ls} = \frac{j_l}{(1 - \epsilon_g - \epsilon_w)\epsilon''_{lf}}. \qquad [11]$$

Østergaard (1965) and others (Kim 1974; Rigby & Capes 1970) calculated ϵ_w by means of [7], [10] and [12],

$$v''_{ls} = \frac{j_l - j_g\epsilon_w\epsilon''_{lf}/\epsilon_g}{(1 - \epsilon_g - \epsilon_w)\epsilon''_{lf}} \qquad [12]$$

instead of [7], [10] and [11]. Equation [12] is [4] with $x = 1$ and the last term neglected. This erroneous neglect of v''_{sf} arose out of failure to balance the upward flow of solids in the wake region of the bed by an equal downward flow of solids in the liquid-fluidized region.

Bhatia (1972) and Bhatia & Epstein (1974) did not limit themselves to the extremities of $x = 0$ and $x = 1$. Instead they wrote a heuristic equation,

$$k = \frac{\epsilon_w}{\epsilon_g} = k''\epsilon^3 \qquad [13]$$

to describe the relative wake holdup, k. This additional equation was required in order to solve for x as an additional unknown in a set of four equations—[3], [6], [7] and [13]. The relationship of k'', the value of k for the solids-free system, to ϵ_g was obtained from the liquid–liquid data of Letan & Kehat (1968) coupled with the value of $k'' = 3.5$ at $\epsilon_g \sim 0$ found by deNevers & Wu (1971) for the conical liquid wake behind a single isolated gas bubble. Bhatia (1972) found that values of x calculated from experimental data were closer to zero than to unity, especially for particles larger than 1 mm and denser than 2.5 g/cm^3. It was therefore tentatively concluded by Bhatia & Epstein (1974), in common with Stewart & Davidson (1964) as well as with Rigby & Capes (1970), that the bubble wakes in three-phase fluidized beds are almost free of solids. The tentativeness of the conclusion arises mainly out of the unverified character of [13].

WAKE HOLDUP CORRELATIONS

Wake holdups evaluated by the procedures described above have been correlated with measured parameters by several investigators (Efremov & Vakhrushev 1970; El-Temtamy 1974; Darton & Harrison 1975; Baker *et al.* 1977; Østergaard 1965). These empirical correlations, written as equations explicit in k, are recorded in table 1 along with an expanded version of [13]. The latter was obtained by noting that the Letan–Kehat data for k'' could be

Table 1. Empirical equations for relative wake holdup

Source	Equation	Assumed x
Østergaard (1965	$k = 0.014\epsilon_g^{-0.5}(j_l - j_{lm})$	1
Efremov & Vakhrushev (1970)	$k = 5.1\epsilon_0^{4.85}\left[1 - \tanh\left(\frac{40j_g\epsilon_0^{10}}{j_l} - 3.32\epsilon_0^{5.45}\right)\right]$	0
Bhatia & Epstein (1974)	$k = \left(0.61 + \frac{0.037}{\epsilon_g + 0.013}\right)\epsilon^3$	$0 \le x \le 1$
El-Temtamy (1974)	$*k = 0.462(j_l/j_g)d_p^{-0.33}$	0
Darton & Harrison (1975)	$k = 1.4(j_l/j_g)^{0.33} - 1$	effectively 0 (see Epstein 1976)
Baker *et al.* (1977)	$k = 1.617(j_l/j_g)^{0.610}\sigma^{-0.654}$	0

*This is a corrected version of the equation appearing in the thesis of El-Temtamy (1974).

adequately represented by

$$k'' = 0.61 + \frac{0.037}{\epsilon_g + 0.013} \qquad [14]$$

when these data are forced through the deNevers-Wu value of $k'' = 3.5$ at $\epsilon_g = 0$.

Wake holdups based on experimental data from several sources (El-Temtamy 1974; Bhatia 1972; Sherrard 1966; Michelsen & Østergaard 1970) were determined on the assumption, based on the aforementioned provisional conclusion of Bhatia & Epstein (1974) and on reported observations of two-dimensional beds (Stewart & Davidson 1964; Rigby & Capes 1970) that $x = 0$ in three-phase fluidization. The results were compared with the correlations of table 1, except for that of Østergaard, which is based on $x = 1$. The comparisons are summarized in table 2 and a sample plot presented in figure 1. All the data tested in this table were in the bubble flow regime.

Despite the considerably different forms of the correlations in table 1, all show k to increase with increasing liquid velocity and hence bed voidage, for a fixed value of gas velocity. Figure 1 illustrates the fact that each also predicts a decrease in k as the gas velocity and hence the gas holdup increases, for a fixed value of liquid velocity. Table 2 shows the correlations of El-Temtamy and of Darton & Harrison to generally give smaller deviations than the others. It is easily seen with reference to table 1 that all the correlations except those of Efremov & Vakhrushev and of Bhatia & Epstein show values of k which approach infinity as gas velocity (and hence gas holdup) approaches zero. The Darton-Harrison equation also gives negative values of k when j_g/j_l exceeds 2.8, as illustrated by figure 1. The equation of Efremov & Vakhrushev was perhaps adversely affected by the fact that the method used by these investigators to measure gas holdup neglected the change in bed level upon introduction of the gas phase. The equation of Baker *et al.*, unlike the others, is based on 2-dimensional bed data (Kim 1974) and on measured values of bubble velocity rather than on [5]. In the light of such

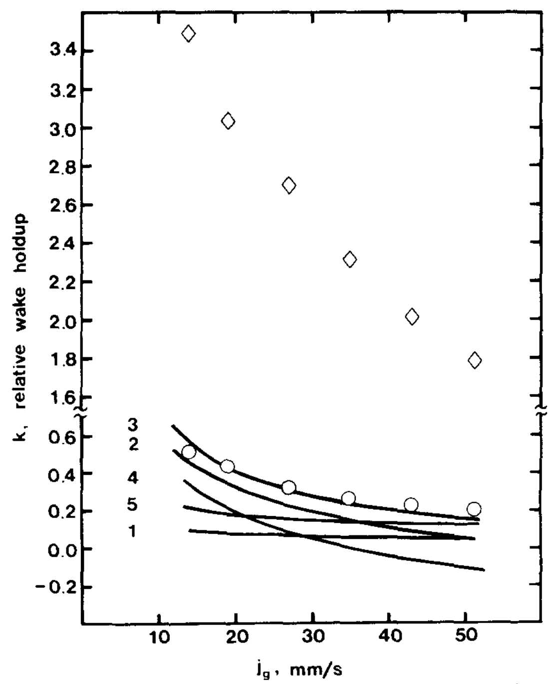

Figure 1. Wake holdup predictions compared with air-water fluidization data of Sherrard (1966) for $d_p = 0.387$ mm, $\rho_s = 2.91$ mg/mm^3, $j_l = 12.4$ mm/sec, $\epsilon_0 = 0.642$. See table 2 for key to solid lines. Circles represent data evaluated assuming $x = 0$, rhombi represent data evaluated by sphere-completing model ($x \neq 0$).

Table 2. Wake holdup correlations compared with data evaluated assuming $x = 0$

Data source	d_p (mm)	ρ_s (mg/mm^3)	j_l (mm/sec range)	j_g (mm/sec range)	No. of data points	R.M.S. Deviation in estimate of k for* 1	2	3	4	5
Sherrard (1966)	0.387	2.91	8–33	14–51	25	0.53	0.51	0.07	0.33	0.40
	1.30	2.96	41–124	14–51	25	1.84	1.19	0.78	1.04	1.70
Michelsen & Østergaard (1970)	1.25	2.67	30–78	15–60	12	0.71	0.32	0.17	0.17	0.55
Bhatia (1972)	0.273	2.938	12–32	19–70	12	0.37	0.44	0.07	0.25	0.17
	0.456	2.935	16–57	20–80	18	0.53	0.60	0.11	0.23	0.29
	0.458	2.578	17–30	1.3–28	30	1.31	0.53	2.00	0.50	0.90
	1.08	2.824	40–68	1.8–62	55	2.19	2.00	2.88	1.74	2.15
	1.08	2.949	76–128	18–66	22	0.66	0.46	0.37	0.14	0.40
	2.18	11.03	63–94	25–112	15	1.34	1.16	0.99	0.83	1.32
El-Temtamy (1974)	0.45	2.529	11–33	13–60	30	0.51	0.26	0.06	0.26	0.47
	0.96	2.930	53–70	13–60	30	1.24	0.88	0.41	0.72	1.21
	2.00	2.936	53–120	13–70	35	2.19	1.91	1.18	1.54	2.18
	3.00	2.930	55–137	13–70	35	1.22	1.14	0.38	0.54	1.25

*1—Empirical equation of Baker *et al.* (1977).
2—Empirical equation of Efremov & Vakhrushev (1970).
3—Empirical equation of El-Temtamy (1974).
4—Empirical equation of Darton & Harrison (1975).
5—Heuristic equation of Bhatia & Epstein (1974).

differences, of the experimental errors in the measurement of ϵ_g and ϵ which underpin the various empirical equations, of the large range of experimental conditions in both the data supporting the respective correlations and in the data tested in table 2, and of the still unverified assumption that $x = 0$ for all conditions, it is hardly surprising that the discrepancies exist.

The equation of Bhatia & Epstein, unlike the others, satisfies the approach to both the limiting conditions of gas–liquid cocurrent flow and a single bubble in a liquid fluidized bed. Thus as $\epsilon_s(= 1 - \epsilon)$ approaches zero (no solids), [13] reduces to $k = k''$; while as j_g and hence ϵ_g approaches zero (single bubbles), the equation reduces to

$$k_0 = 3.5\epsilon_0^3. \quad [15]$$

A test of [15] against several of the data examined in table 2, again assuming $x = 0$, is performed by first plotting k vs j_g for different values of j_l, as exemplified by figure 2. The intercepts at $j_g = 0$ were then plotted as k_0 vs ϵ_0 in figure 3, ϵ_0 being the liquid holdup at zero gas flow. Figure 3 shows that the exponent 3 in [15] and therefore also [13] represents most of the experimental data quite well, but that the coefficient modifying ϵ_0^3 varies widely on both sides of 3.5 and appears to increase with increasing particle size in each investigation considered.

In view of these results, it would appear that neither the holdup nor the inter-related solids content of the wakes behind the bubbles in three-phase fluidized beds can be ascertained with confidence by existing methods. A more fundamental approach is called for, whereby k can be estimated without any prior assumption about x, and such an approach is presented below.

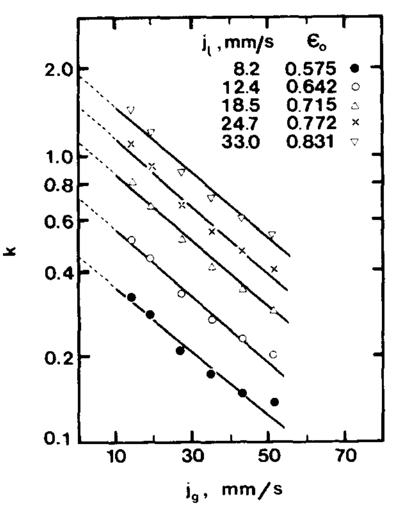

Fig. 2.

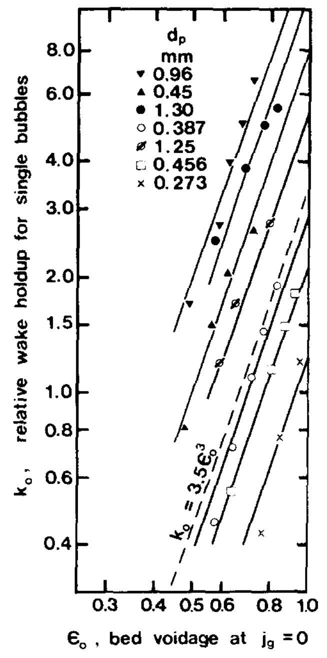

Fig. 3.

Figure 2. Relative wake holdups calculated assuming $x = 0$ from air–water fluidization data of Sherrard (1966) for $d_p = 0.387$ mm, $\rho_s = 2.91$ mg/mm^3.

Figure 3. Test of [15] assuming $x = 0$. Data of: El-Temtamy (1974) for 0.96 and 0.45 mm glass beads, Sherrard (1966) for 1.30 and 0.387 glass beads, Michelsen & Østergaard (1970) for 1.25 mm glass beads, Bhatia (1972) for 0.456 and 0.273 glass beads.

SPHERE-COMPLETING WAKE MODEL

It is assumed, after studies of gas–liquid systems (Davies & Taylor 1950; Collins 1965; Hills 1975), gas fluidized beds (Rowe & Partridge 1962; Grace 1970) and gas–liquid fluidized beds (Stewart & Davidson, 1964; Henriksen & Østergaard 1974; Darton & Harrison 1976), that the sphere-completing volume of spherical cap bubbles, which predominate over the smaller spherical and ellipsoidal bubbles with which they co-exist in 3-dimensional gas–liquid fluidized beds, is the effective volume of the bubble wakes. The 2-dimensional analogue of a spherical cap bubble is a circular- or elliptical-cap bubble (Grace 1970; Hills 1975). The included angle, θ, of the caps is in general a function of bubble Reynolds No. (Grace 1970) but over the range of conditions commonly encountered in air–water fluidization, at least for particles of 2.5–3.0 specific gravity up to 2 mm in diameter, Henriksen & Østergaard (1974) found θ to be nearly independent of bubble size and uniquely dependent on kinematic viscosity of the medium surrounding the bubble. The viscosity μ_0 of a liquid–solid fluidized bed, which is the medium in the case of three-phase fluidization, can be estimated from one of three relationships presented by Henriksen & Østergaard from the literature (Hetzler & Williams 1969; Rigby *et al.* 1970; Trawinski 1953), depending on the range of particle size involved. The density of such a bed is

$$\rho_0 = \rho_l \epsilon_0 + \rho_s(1 - \epsilon_0) \qquad [16]$$

where ρ_l and ρ_s are the densities of the liquid and the solids, respectively, while the kinematic viscosity is μ_0/ρ_0. Knowing this kinematic viscosity, θ can be read off from figure 3 of Henriksen & Østergaard (1974), ignoring any differences in θ as between 2-dimensional and 3-dimensional beds (Grace 1970). The wake–bubble volume ratio, k_0, of a single bubble can then be determined by geometry.

To relate k_0 for a single bubble to k for a multi-bubble system, liquid–liquid data of Yeheskel & Kehat (1973) deemed to be more accurate than the earlier data of Letan & Kehat (1968) are invoked. Two sets of such data are plotted in figure 4 for drop holdups less than 0.35. These semi-log plots are both well correlated by

$$k'' = k_0'' e^{-5.08\epsilon_d} \qquad [17]$$

where k_0'' is the intercept at $\epsilon_d = 0$ and agrees well with data on single drops (Yekeskel & Kehat 1971). Interestingly, the two liquid–liquid values of k_0'' in figure 4 also closely bracket the 3.5 of deNevers & Wu (1971) for gas–liquid systems. Assuming that

$$\frac{k}{k_0} = \frac{k''}{k_0''} \qquad [18]$$

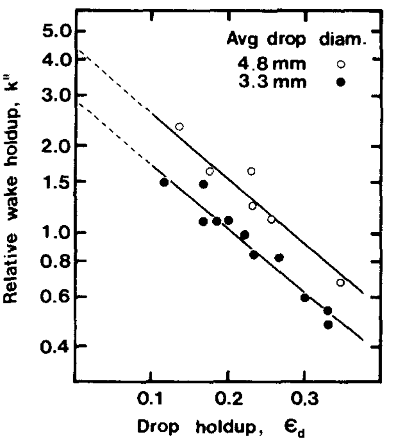

Figure 4. Relative wake holdups for kerosene drops dispersed in water, data of Yeheskel & Kehat (1973).

and that wakes behind bubbles behave similarly to those behind drops, one can then write

$$k = k_0 e^{-5.08\epsilon_g} \qquad [19]$$

and thus relate k_0 to k, knowing ϵ_g for the three-phase fluidized bed. Knowing also j_g, j_l, ϵ, n and v_1, the value of x, which characterizes the solids composition of the wakes, can then be determined by simultaneous solution of the generalized wake equations [3], [6] and [7] for the unknowns ϵ''_{lf}, v''_{ls} and x.

The procedure for 3-dimensional gas–liquid fluidized beds can be summarized as follows:

(1) Estimate the kinematic viscosity of the fluidized bed at the given liquid flow and zero gas flow from [13]–[15] of Henriksen & Østergaard (1974) for μ_0, and [16] above for ρ_0.

(2) Read off θ from figure 3 of Henriksen & Østergaard, whence evaluate k_0 from the geometry of spherical-cap bubbles.

(3) Determine k and hence $\epsilon_w(= k\epsilon_g)$ from [19].

[4] Solve [3], [6] and [7] for x.

This procedure was applied to data from the studies referred to in table 2 for particles up to 2 mm in diameter and specific gravity in the range 2.5–3.0 (Sherrard 1966, Michelsen & Østergaard 1970; Bhatia 1972; El-Temtamy 1974), as well as to the 2-dimensional data of Kim (1974) for 1 mm, 2.95 specific gravity particles fluidized by air plus various liquids more viscous than water up to 6.3 cP. For the 2-dimensional data, the geometry of a circle rather than a sphere was used to calculate k_0 from θ. Unlike the data of table 2, about 20% of the 3-dimensional data thus treated were in the slug flow regime.

RESULTS

As illustrated in figure 1, values of k determined by the above procedure were considerably in excess of those based on the assumption that $x = 0$, in the case of particles much smaller than 1 mm. For 1 mm and larger particles, however, the values of k by this model were in good agreement with those based on $x = 0$, at least with water as the fluidizing liquid. In general k increased with decreasing d_p (at constant ϵ) and with increasing j_l and hence ϵ (at constant d_p), as a consequence of decreasing bed viscosity and hence θ in both cases. It also decreased with increasing j_g and hence ϵ_g, as a consequence of [19].

The relative wake solids content, x, was found to increase with decreasing particle size, increasing gas velocity and increasing liquid viscosity. These results suggested a correlation of the form

$$x = a - b\frac{v_1}{v} \qquad [20]$$

or alternatively

$$x = 1 - b'\frac{v_1}{v} \qquad [21]$$

or possibly even

$$x = 1 - \frac{v_1}{v} \qquad [22]$$

where v is either the average velocity of the gas bubbles relative to the column walls, v_g, given by [5], or the velocity of the gas relative to the total liquid,

$$v_{gl} = \frac{j_g}{\epsilon_g} - \frac{j_l}{\epsilon_l} \qquad [23]$$

or the velocity of the gas bubbles (and their wakes) relative to the liquid in the liquid-fluidized region,

$$v_{glf} = \frac{j_g}{\epsilon_g} - \frac{j''_{lf}}{\epsilon''_{lf}}. \quad [24]$$

Least squares correlations of the nine possible resulting combinations are recorded in table 3.

It is seen in table 3 that the correlation coefficient of x with v_{gl} is significantly larger than with v_g, but only slightly smaller than with v_{glf}. Table 3 also shows the standard deviations by [21] to be significantly lower than those by [22] but only marginally higher than those by [20]. Since logic would indicate an upper value of a equal to unity, and since v_{gl} is considerably easier to determine than v_{glf} (which can only be calculated after x is known), the recommended combination is [21] with $v = v_{gl}$, giving

$$x = 1 - 0.877\frac{v_1}{v_{gl}} \quad [25]$$

with a standard deviation in estimate of x of ± 0.101, as shown in figure 5. Forcing the correlation through $x = 1$ at $v_1/v_{gl} = 0$ is further vindicated by the fact that the least squares value of a in [20], with $v = v_{gl}$, is 1.014.

Other things being equal, the value of x increases as d_p decreases (as observed by Rigby & Capes 1970) and as μ_l increases. This is because the terminal velocity, v_1, decreases with decreasing d_p and with increasing μ_l, and the change in x then follows from [25]. Similarly x increases as v_g increases due to the corresponding increase in v_{gl}, the effect of which on x again follows directly from [25]. The effectiveness of v_{gl} in opposing the influence of v_1 is applicable only to particles smaller than about 1 mm, for solids of specific gravity less than 3, with water as liquid. For somewhat larger and/or heavier particles, v_1 dominates (especially at low gas velocities) and $x \simeq 0$, unless μ_l increases appreciably. As d_p and /or ρ_s increase still further, [25] eventually gives negative values of x (at $v_1/v_{gl} > 1.14$) and therefore becomes inapplicable. In this region, however, bubble wakes play a diminishing role in the bed dynamics (Bhatia & Epstein 1974).

Table 3. Regression analysis of x vs v_1/v

v	v_g	v_{gl}	v_{glf}
Correl'n coeff.	−0.863	−0.906	−0.920
	[20]: $x = a - b\frac{v_1}{v} \pm S_1$		
a	1.086	1.014	0.966
b	1.444	0.909	1.090
S_1	0.120	0.101	0.093
	[21]: $x = 1 - b'\frac{v_1}{v} \pm S_2$		
b'	1.216	0.877	1.167
S_2	0.125	0.101	0.095
	[22]: $x = 1 - \frac{v_1}{v} \pm S_3$		
S_3	0.146	0.117	0.113

MECHANISM OF WAKE SOLIDS ENTRAINMENT

Equation [25] can be rationalized by assuming that a gas bubble initially collects a wake from the liquid-fluidized zone with a solids content equal to that of this zone. As the bubble rises, particles tend to settle out of the wake depending on their terminal velocity v_1, which increases with increasing d_p and decreasing μ_l. Counteracting this tendency is liquid circulation in the wake, which is fostered by the gas–liquid relative velocity, v_{gl}. (This circulation is presumed to act in the same manner as a mechanical impeller with a horizontal shaft in a beaker of water and settled sand: despite the fact that there is no net upward velocity, some sand is

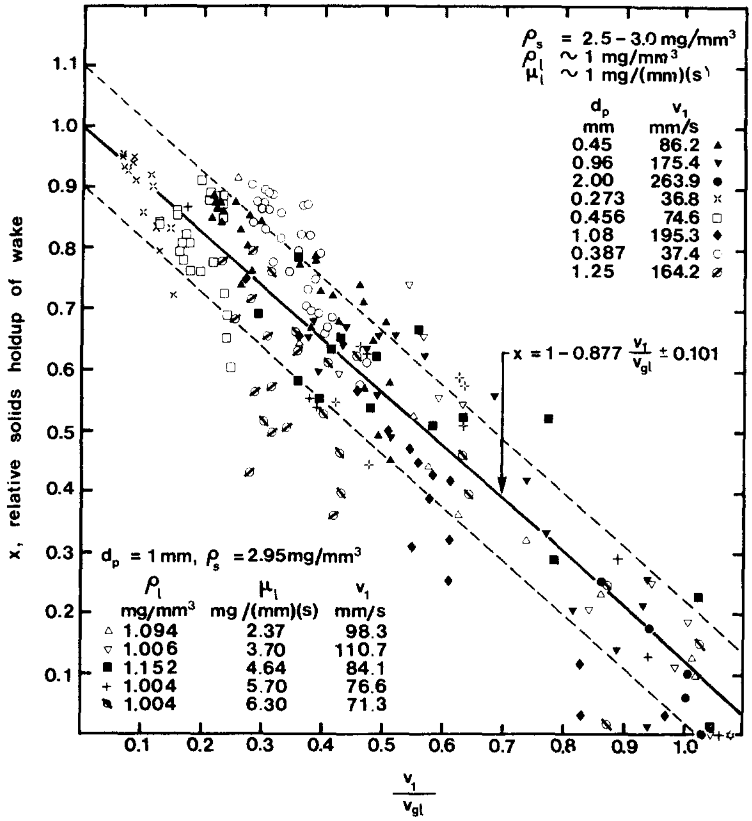

Figure 5. Correlation for x obtained via sphere-completing wake model in conjunction with generalized wake equations. Data of: El-Temtamy (1974) for 0.45, 0.96 and 2.00 glass beads, Bhatia (1972) for 0.273 and 0.456 glass beads, Sherrard (1966) for 0.387 glass beads, Michelsen & Østergaard (1970) for 1.25 mm glass beads, Kim (1974) for 1.0 mm glass beads. Total No. of data points = 170.

kept suspended by lift forces induced by the circulation). The process repeats itself as the wake is shed and a new wake formed. The average concentration of solids in the wake is then dependent on the frequency of wake shedding, f, which commences at bubble Reynolds numbers as low as 80 (Narayanan *et al.* 1974). The greater this frequency the higher is x, which is a space-time-average value.

The wake shedding frequency, f, is usually non-dimensionalized as a Strouhal number,

$$Sr = \frac{D_b f}{v_{gl}}. \quad [26]$$

For a given value of Sr, an increase in v_{gl} increases f and hence x. Thus v_{gl} plays its role in increasing x both by increasing internal circulation and by increasing f. The apparent effect via [26] of increasing the bubble diameter, D_b, is to decrease f. However, in the bubble regime often encountered in three-phase fluidization, Sr increases with bubble Reynolds number to a power in excess of unity (Lindt & de Groot 1974), so that the net effect of increasing D_b would be to increase f and hence x. This effect on x is reinforced by an accompanying increase in bubble rise velocity (Massimilla *et al.* 1961; Rigby *et al.* 1970; Darton & Harrison 1974). Such an increase in the solids content of wakes with increasing bubble diameter has in fact been observed qualitatively by Page (1974). Since under certain conditions the bubble diameter grows as the particle diameter decreases (Østergaard, 1971), small d_p and large D_b would act in concert under these conditions to enhance any tendency towards wake entrainment of solids.

CONCLUDING REMARKS

Bhatia (1972) has observed excessive entrainment and even elutriation of particles smaller than 1 mm and lighter than 3 mg/mm^3 in very viscous liquids. The expansion rather than contraction response of his 1 mm, 2.8 mg/mm^3 spheres fluidized by a 63-cP liquid plus air, on introducing the air, may be caused by the fact that x via [25] is very close to unity for this system. The applicable equations [7], [10] and [11] then predict a bed expansion on increasing j_g. [The alternate explanation of a wakeless bubble offered by Bhatia (1972) and Epstein (1976) was based on a theoretical treatment by Levich (1962) which is at odds with reality for the bubble Reynolds number involved].

Page & Harrison (1974) have reported that particle entrainment from the surface of three-phase fluidized beds increases as j_l and d_p decrease and as j_g and D_b increase.

These results are all qualitatively in agreement with [25] and the proposed mechanism to explain this equation. In a future paper, [25] will be applied to the quantitative prediction of entrainment in the freeboard of three-phase fluidized beds.

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Postscript—Both Rigby & Capes (1970) and Darton & Harrison (1975, 1976) have observed the bubble wakes in 2-dimensional three-phase fluidized beds to be comprised of an upper region of particle-free liquid and a lower region of particles and liquid. This observation lends strong support to the mechanism of wake solids entrainment and settling described above.

NOMENCLATURE

- a constant in [20];
- b constant in [20];
- b' constant in [21];
- D_b bubble diameter, mm;
- d_p particle diameter, mm;
- f wake shedding frequency, l/sec;
- j_g superficial gas velocity, mm/sec;
- j_l superficial liquid velocity, mm/sec;
- j_{lm} minimum superficial liquid fluidization velocity, mm/sec;
- j''_{lf} superficial velocity in the liquid-fluidized zone, mm/sec;
- k relative wake holdup $= \epsilon_w/\epsilon_g$;
- k_0 relative wake holdup for single bubbles;
- k'' relative wake holdup for gas–liquid or liquid–liquid; systems $= \epsilon_w/\epsilon_d$ for liquid–liquid system;
- k''_0 relative wake holdup for single bubbles or drops in solids-free liquid media;
- n exponent in Richardson–Zaki type equation for liquid fluidization;
- R.M.S. root mean square;
- S_1, S_2, S_3 standard deviations in estimate of x via [20], [21] and [22] respectively;
- Sr Strouhal number $= D_b f/v_{gl}$;
- v gas velocity, frame-of-reference unspecified, mm/sec;
- v_g average velocity of gas $= j_g/\epsilon_g$, min/sec;
- v_{gl} velocity of gas relative to liquid, mm/sec;
- v_{glf} velocity of gas relative to liquid in liquid-fluidized zone, mm/sec;
- v''_{ls} velocity of liquid relative to solids in liquid-fluidized zone, mm/sec;
- v''_{sf} solids velocity in liquid-fluidized zone, mm/sec;

v_i j_l/ϵ_0 as ϵ_0 approaches unity for liquid fluidization, mm/sec≡terminal velocity of single particles in liquid velocity field of fluidization column, mm/sec;
x relative solids holdup of wake $= \epsilon''_{sw}/\epsilon''_{sf}$;
ϵ bed voidage $= \epsilon_l + \epsilon_g = 1 - \epsilon_s$;
ϵ_0 bed voidage for zero gas flow, i.e. for liquid fluidization;
ϵ_d drop holdup = dispersed phase holdup;
ϵ_g gas holdup = volume fraction of gas in bed;
ϵ_l liquid holdup = volume fraction of liquid in bed;
ϵ_s solids holdup = volume fraction of solids in bed;
ϵ_w wake holdup = volume fraction of wake zone;
ϵ''_{lf} liquid holdup in liquid-fluidized zone;
ϵ''_{sf} solids holdup in liquid-fluidized zone;
ϵ''_{sw} solids holdup in wake zone;
θ included angle of spherical- or circular-cap bubbles;
μ_l viscosity of liquid, mg/(mm) (sec)≡centipoises;
μ_0 viscosity of liquid fluidized bed, mg/(mm) (sec);
ρ_l density of liquid, $mg/mm^3 \equiv g/cm^3$;
ρ_0 density of liquid fluidized bed, mg/mm^3;
ρ_s density of solids, mg/mm^3;
σ gas–liquid surface tension, $dynes/cm \equiv g/sec^2$.

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